Calculate the Shapes of Neutron Star Cores

Detailed Response to the caught zomb from *poly*
As described at www.zombal.com
Report by *dragozzine*
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Zomb Statement:

Neutron stars and pulsars are believed largely made up of very densely-packed, spinning cores of neutrons. Calculations are wanted for the shapes of these spinning cores.

The default assumption is that such stars are spherical, but in view of the forces involved, this assumption is unlikely. Because of the mass concentration in these cores, gravitational forces are very high, but because of their rapid rates of spin, centrifugal (centripetal) forces are also very high.

In the attached sketch 'Neutron Core.jpeg', some possibilities are shown. Even in the relatively slowly-rotating Earth, this rotation is enough to swell it out at the equator into an oblate spheroid (B). The interplay of gravitation and centrifugal forces in star cores might lead to a bobbin shape (D), a spindle shape (E), a discoid (C), a torus, or a hollow rotating cylinder, or something else.

According to http://en.wikipedia.org/wiki/Neutron_star, a typical neutron star has a mass between about 1.4 and 3.2 solar masses (with a calculated radius of about 12 km if spherical), and rotation periods from about 1.4 ms to 30 seconds.

Shape calculations are wanted for this range of masses and rotation periods. Distorting effects of magnetic fields, relativistic influences, etc, may be noted but ignored in calculations.

Response

Abstract: A background discussion of neutron stars and the shapes of rapidly rotating objects is given. By comparing the rotational force pushing the equator outward to the gravitational force pulling the equator inward, we calculate the standard quantity used to determine the importance of rotation on shape. In the most extreme neutron star case, we see that this is about w=0.1, which is too small for any exotic shapes and suggests that all neutron stars are oblate spheroids. A couple of references are also pointed out, showing that astronomers are familiar with these effects and that they are generally too small to be important for most investigations.

Background

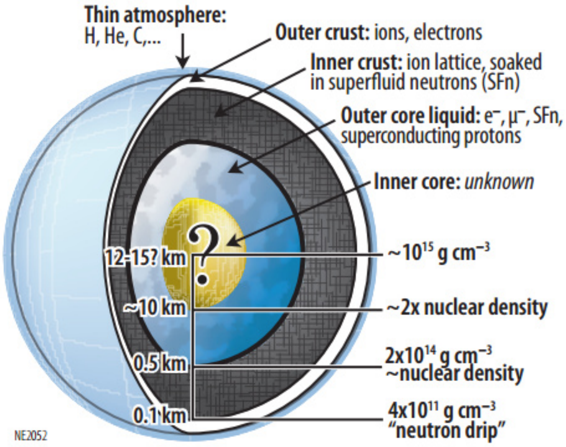
First, some basic astrophysics definitions and background.

Neutron stars are the remnants of stars that have effectively collapsed under their own weight and “died”. It is effectively a giant nucleus, composed almost entirely of neutrons and held up by neutron “degeneracy pressure”. Neutron stars, if they could grow in size, could overcome this pressure at which point they would collapse into a black hole.

Highly magnetized neutron stars can emit powerful radio beams in preferential directions, similar to a light house, and when these beams are pointed towards our telescopes, the neutron stars are also called “pulsars”. As pointed out, based on various astronomical observations, we know the basic properties of neutron stars, such as mass, radius, and rotation.

These can be used to estimate the interior structure and shapes of such neutron stars. First, let me clarify what I think is meant by neutron star “core”. It is generally the core of the star that becomes a neutron star and the remaining atmosphere is blown off very quickly. This atmosphere forms a “supernova remnant” or “planetary nebula” that is ejected and separate from the neutron star itself. These are often quite “pretty” looking, as in the Crab nebula, which surrounds the very small neutron star pulsar. This atmosphere is not considered to be part of the neutron star.

The interior properties of neutron stars are not well known since the properties of matter at the ultraextreme pressures and densities is not understood. Shown is a figure from a July 2012 report on “NICER” the Neutron star Interior Composition ExploreR, a proposed instrument to fly on the space station to understand more about the insides of these fascinating objects (http://heasarc.gsfc.nasa.gov/docs/nicer/papers/NICER-SPIE-July2012-v4.pdf).



It has been inferred that neutron stars have a “core” which is defined separately from the rest of the object by the fact that the properties reach a density where the neutrons form a superfluid and the small number of residual protons form a superconductor. The shape of the core of the neutron star can be addressed with the calculations shown here, but it is more complicated because of the presence of additional layers. Here, we will focus on describing the shape of the neutron star as a whole, which seems to be the intent of the Zomb description.

In order to be clear, we will refer to the neutron star “core” if we are talking specifically about the internal portion of the neutron star (as shown in the figure) and will redefine the question as asking about the shape of rotating neutron stars themselves, i.e., the whole object, but not including the blown-off atmosphere seen in the supernova remnant of planetary nebula (which is not connected directly to the neutron star anyway, and whose shape is an entirely different question).

The shape that an object takes due to rotation depends on its properties and interior structure. In fact, you can deduce from the different shapes of Jupiter and Saturn, that the latter has a large core (in proportion to the size of the planet) while the former does not.

In Ragozzine & Wolf 2009 (http://arxiv.org/abs/0807.2856), we explain some of the calculations behind the "theory of figures" as it is called, which were developed by many previous authors. There we focused on the "slow" rotation regime (i.e., treating only first-order effects), but due to my work on Haumea (Brown et al. 2007, Ragozzine & Brown 2007, Ragozzine & Brown 2009), I'm also very familiar with what happens at higher rotation rates (Maclaurin spheroids and Jacobi ellipsoids, etc.).

When rotation is added to a spherical object, it begins to bulge at the equator. This is called an “oblate” or Maclaurin spheroid. If you were to cut through the objects North and South poles (i.e., with a plane that contains the axis of rotation), you would find an ellipse. The deviation of this ellipse from a circle is called the “eccentricity.” At low rotation rates, the ellipse is the same shape no matter where you cut (i.e., there is no different in the ellipse as a function of “longitude”). This can also be explained by cutting the object along latitude lines, perpendicular to the axis of rotation, and seeing that all such slices of the object are perfectly circular. Thus, an oblate spheroid can be described with a “polar radius”, from the center to the North/South pole, and an equatorial radius, from the center to anywhere on the equator. The equatorial radius is always a little larger than the polar radius and the difference between these two, divded by the equatorial radius, is called the “flattening”.

Increasing the spin of an oblate sphere, there will eventually be too much angular momentum and the object instantly extends along one axis to become a prolate or Jacobi ellipsoid. This is a triaxial ellipse: cutting the object in the three possible perpendicular directions through the center will each give an elliptical shape. It is somewhat similar to a cigar shape, but not as long as a typical cigar. At much higher rotation rates, "discoids" are possible, depending on the properties of the material. Uusally, you eventually reach the point where there is too much angular momentum and the object undergoes fission and becomes two tidally-locked objects (sometimes called a "Roche binary"). After that point, you can add more angular momentum and you just make the orbit of the binary larger and larger without producing any additional spin.

The quantity that tells you roughly where you are in this continuum of shape possibilities is the ratio of the centrifugal force to the gravitational force, both measured at the equator. The centrifugal force is the force that is causing the object to bulge out at the equator, so you can imagine that if this was a significant fraction of the gravity felt by an object at the equator, the bulge would be very big. For our slow spinning Sun, this is a very small amount (something like 0.00001 or so), for the Earth it's something like 0.3% and like 3% for Jupiter and Saturn. Let's call this ratio *w.*

We can calculate this ratio for a neutron star by using equations for the two forces and dividing to get

$$w = \frac{4\pi^2 R^3}{GMP^2}$$

where pi is 3.14..., R is the radius of the neutron star, G is the universal gravitational constant, M is the mass of the neutron star, and P is the period of rotation. Plugging in the most extreme values possible for a neutron star (2 solar masses, 12 km of radius, and period of 1.4 milliseconds), we find that w = 0.13 at the most extreme. I think in real neutron stars, there is a relationship between spin rate and size such that this most extreme value is not actually achieved. When the spin rate goes down just a little bit, to a period of 10 ms, then w drops substantially to less than a percent (since it is period squared that matters).

Despite their incredible spin rates of rotating once every millisecond, with w <= 10%, they fall into the regime of Maclaurin spheroids; the transition to Jacobi ellipsoids is around w = 30-50%, depending on the properties of the object. Based on some initial looking around, the effects that we have ignored (general relativity, magnetism, higher-order distortions, etc.) appear to be unimportant for this general picture.

The next step would be to use this value of w to estimate the eccentricity of the oblate spheroid or the flattening. However, this requires having a detailed description of the interior density distribution of a neutron star, which is also not known. I can say that the flattening and eccentricity are typically similar to the value of w, i.e., the polar radius will be roughly 10% smaller than the equatorial radius for the most extreme neutron star cases.

It looks like the deformation and shape of neutron stars has been investigated, including many more relevant details that I have mentioned here. Here is one of several examples: **http://iopscience.iop.org/1742-6596/337/1/012021/**

In particular, neutron star astronomers appear to be aware of the possibilty that spin can contribute to their understanding of neutron stars; it is not a forgotten aspect of the problem, but one that can usually be ignored. For example, the effect of non-spherical shape on the orbit of a pulsar-pulsar binary is known to be negligibly small (http://www.ns-grb.com/PPT/Lattimer.pdf).

I believe that the above answers the questions posed in this Zomb, along with given significant explanation. Please let me know if any of the above arguments are unclear or if you have any other questions or zombs. Doing a Google or Wikipedia search on most of the above concepts will provide helpful insights and references. Other bibliographic citations are available upon request.

Thank you for this opportunity.

Dr. Ragozzine is a Postdoctoral Researcher in the Astronomy Department at the University of Florida. He is currently funded by the United States National Aeronautics and Space Administration (NASA) to work on the NASA Kepler Space Mission. This document is his personal composition and reflects his personal opinion, not that of the University of Florida, NASA, or any other party. He has many years of research experience in the orbital dynamics of planetary systems, the Kuiper belt within our own solar system, and extra-solar planets observed around other stars.