My answer comes from direct and immediate experience. I was busy last semester teaching classes. One of the classes I taught was graduate-level planetary interiors, in which we go through this calculation in great detail. It's very safe to say that I'm an expert on this question.

You bring up a good point about the evolution of gravity as a function of the internal density profile. You mentioned the very old result that the gravitational attraction of a sphere can be approximated as all of the mass being concentrated at the central point. The derivation of that result, which uses a calculus-based evaluation similar to the one you suggest, actually starts with a different calculation. The first calculation is that the gravitational field of an (infinitesimally-thin) spherical SHELL of matter is equal to 0 on the inside and on the outside is exactly equal to the gravitational field of a point mass at the center of a shell with the mass of the whole shell. Newton's result then follows by dividing up the sphere into an infinite number of thin shells and adding (integrating) up all the mass of all of them. Note that many of these calculations and results are explained in detail in a previous response to your "Background Gravitation" Zomb that I sent about a year ago.

The interesting aspect of this derivation is that you can that say that the gravitational field of a sphere is the same as if all the mass were at the center, IRRESPECTIVE of the internal density profile. No matter what the interior density profile is, all the mass of all the shells is "displaced" as it were to the center of the body. So, the gravitational field of a heterogenous (but still spherically-symmetric) sphere is described by the well-known equation and result. The only thing you would need to check is that if you add up the mass of very thin spherical shells in the PREM model that you get the total mass of the Earth. But I can assure you this is the case.

I have worked on (and have students working on) similar problems for exoplanets (see Ragozzine & Wolf 2009, Maxwell, Ragozzine, et al. 2010). Note that the PREM model that you refer to (which is the standard for the Earth, even today), actually gives the "surface gravity" or gravitational coefficient as a function of distance within the Earth in the last column as "g". I can tell you from experience that the way PREM is calculated already takes into account all the calculations I've mentioned above. Of course, at the surface, the total surface gravity matches what is expected. That is, PREM by definition CANNOT find a density distribution inside the Earth that violates the measured surface gravity.

I think this should answer your question, but I'm happy to discuss a few more details if you'd like me to or provide additional evidence. Great question that hopefully I have satisfactorily answered.

Darin Ragozzine