Mass Lost by the Earth

Calculation by Daniel Branscombe

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Given Values:

G: Geothermal flux of the Earth, -0.08 $\frac{\frac{J}{s}}{m^2}$

S: Surface area of the Earth, $5.10 \times 10^8 km^2 = 5.10 \times 10^{14} m^2$

Y: 1 year in seconds, $365.25 days \times \left(\frac{24hr}{day}\right) \times \left(\frac{60min}{hr}\right) \times \left(\frac{60s}{\min}\right) = 3.15576 \times 10^7 s$

C: speed of light, $\frac{299{,}792{,}458m}{s}$

Calculations: (using M for mass to avoid confusion with m for metres)

$E = Mc^2$ taking the derivative in terms of time (in seconds)

$\frac{dE}{dt} = \frac{dM}{dt} c^2$ now the change in energy is given by the flux times the surface area

$GS = \frac{dM}{dt} c^2$ solving for the change in mass

$\frac{dM}{dt} = \frac{GS}{c^2}$ inputting our given values we get

$$M' = \frac{dM}{dt} = \frac{GS}{c^2} = \frac{\left(-\frac{0.08J}{sm^2}\right) * (5.10 \times 10^{14} m^2)}{(\frac{299792458m}{s})^2} = -4.5396 \times 10^{-4} kg/s$$

Thus the Earth is losing $4.5396 \times 10^{-4} kg/s$.

To convert this to how much is lost in one year we have

$$Y * M' = 3.15576 \times \frac{10^7 s}{year} * -4.5396 \times \frac{10^{-4} kg}{s} = -1.43259 \times 10^4 kg/year$$

Thus in one year, the Earth loses approximately 14,000 kg of mass.

Over a span of 1 billion years, the Earth would lose $1.43259 \times 10^{13} kg$.

The Earth's current mass is $5.98 \times 10^{24} kg$ [http://pds.nasa.gov/planets/special/earth.htm]

Thus, as a percentage of the current mass, the loss over 1 billion years is:

$\frac{1.43259 \times 10^{13} kg}{5.98 \times 10^{24} kg} = 2.3956 \times 10^{-12}$ or approximately 2.4×10^{-10} percent.

Other Considerations:

What has not been considered in the above calculation is the amount of surface area that is lost due to the loss of mass. Now surface area is given by $S = 4\pi r^2$ where r is the radius of the Earth. Now the mass of the Earth is given by $M = D * V$ where D is the density of the Earth and V is the volume of the Earth. From our known values of S and M we can derive that $D = 5522 kg/m^3$. Using the equation for the volume of a sphere, we have

$V = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi\left(\frac{S}{4\pi}\right)^{3/2} = \frac{M}{D}$ solving for S we get

$S = 4\pi\left(\frac{3M}{4\pi D}\right)^{2/3}$ using our equation from above we have

$\frac{dM}{dt} = \frac{GS}{c^2} = \frac{4\pi G}{c^2}\left(\frac{3M}{4\pi D}\right)^{2/3}$ which gives us a nice ODE for M, solving it for M we have

$$\frac{c^2}{4\pi G}\left(\frac{3M}{4\pi D}\right)^{-\frac{2}{3}} dM = dt$$

$\frac{c^2}{4\pi G}\left(\frac{3}{4\pi D}\right)^{-\frac{2}{3}} M^{-\frac{2}{3}} dM = dt$ Integrating both sides gives

$$\frac{3c^2}{4\pi G}\left(\frac{3}{4\pi D}\right)^{-\frac{2}{3}} M^{1/3} = t + k$$

We know that when $t = 0$ $M = M_e$ the initial mass of the Earth, thus

$k = \frac{3c^2}{4\pi G}\left(\frac{3}{4\pi D}\right)^{-\frac{2}{3}} {M_e}^{1/3}$ substituting above and solving for M we get

$$M = \frac{4\pi G^3}{3c^6 D^2}\left(\frac{3c^2}{4\pi G}\left(\frac{3}{4\pi D}\right)^{-\frac{2}{3}} {M_e}^{1/3} - t\right)^3$$

Using our known values, after 1 year, the mass lost agrees with the initial calculation to at least 1 decimal. Projecting out 1 billion years, this knew calculation is approximately $11kg$ less than the previous calculation amount. Thus the simpler initial method can be deemed accurate enough for most purposes.