Eccentricity Vs. Radiation

Here we show the percent change in the sun's radiation as a function of the eccentricity of the Earth's orbit about the sun. Because the Earth's orbit is an ellipse, at different points in the orbit, the Earth is at a changing distance from the sun. This means that the amount of radiation from the sun's rays captured by the Earth changes. Based purely on the distance between the Earth and the Sun, the radiation level will be maximum when the Earth is at the perihelion and minimum, when the Earth is at the aphelion of the orbit.

In order to calculate the percent change, we first note the relation between the eccentricity and the ratio between the perihelion and the aphelion. We have

$$\frac{r_p}{r_a} = \frac{1-e}{1+e}$$

where r_a is the aphelion, i.e. the farthest distance from the Sun, r_p is the perihelion, i.e. the closest distance from the Sun, and e is the eccentricity. Now, the inverse square law says that the amount of radiation will decrease in proportion to the square of the distance. So we have that the radiation at r_a is proportional to r_a^{-2} and at r_p it is proportional to r_p^{-2}. Thus, the radiation at r_a is $(r_p/r_a)^2$ of the radiation at r_p.[1] For example, when $r_p = r_a$ when $e = 0$, the radiation levels are the same and the difference is zero. Therefore, the difference in radiation levels at the aphelion and perihelion is

$$1 - \left(\frac{r_p}{r_a}\right)^2.$$

We can also represent this in terms of the eccentricity of Earth's orbit:

$$1 - \left(\frac{1-e}{1+e}\right)^2.$$

The Earth's eccentricity varies between .0034 and .0058 and below we plot the percentage difference in radiation levels for these eccentricity values.

1. Note that $r_p \leq r_a$ so that $(r_p/r_a)^2 \leq 1$.

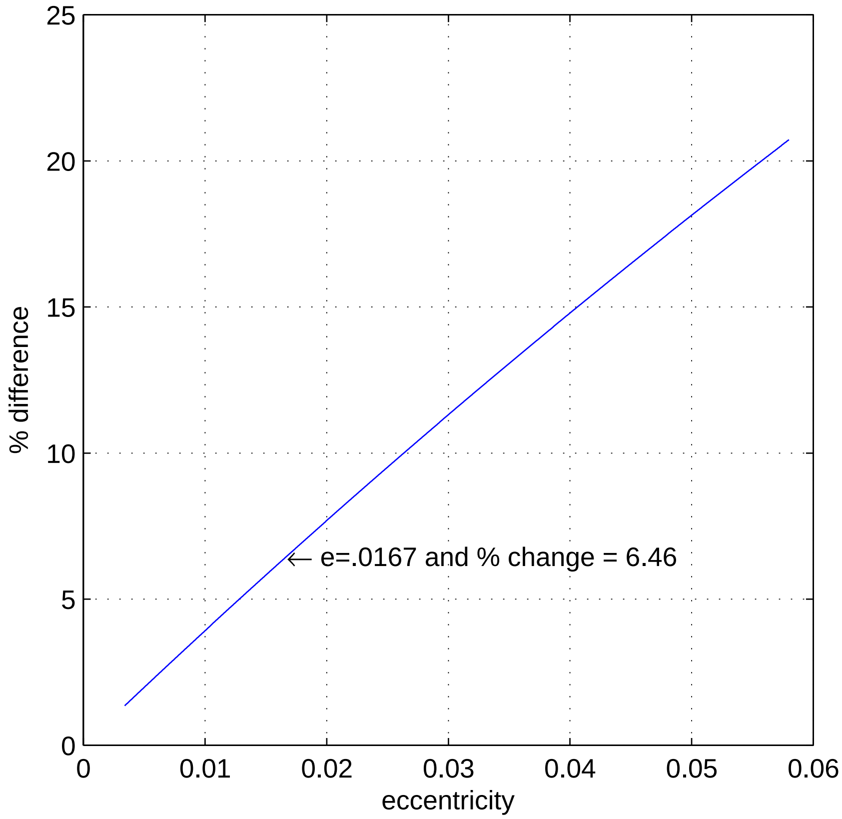

Figure 1. Percentage difference between radiation levels at the aphelion and the perihelion for difference eccentricities of Earth's orbit. Currently, we are at eccentricity .0167, which means we obtain 6.46 % more radiation at the perihelion than at the aphelion.

We also list a table of values between $e = .0034$ and $e = .0058$.

	e	% diff
1	0.0034	1.3508
2	0.0057	2.2445
3	0.0080	3.1300
4	0.0102	4.0076
5	0.0125	4.8773
6	0.0148	5.7392
7	0.0171	6.5933
8	0.0193	7.4397
9	0.0216	8.2785
10	0.0239	9.1098
11	0.0262	9.9337
12	0.0284	10.7502
13	0.0307	11.5594
14	0.0330	12.3613
15	0.0353	13.1561
16	0.0375	13.9439
17	0.0398	14.7246
18	0.0421	15.4984
19	0.0444	16.2653
20	0.0466	17.0254
21	0.0489	17.7787
22	0.0512	18.5254
23	0.0535	19.2655
24	0.0557	19.9990
25	0.0580	20.7261

Figure 2. Eccentricity versus % difference in radiation levels.

For reference, we show that current Earth's orbit overlaying the perfectly circular orbit. We assume that the Sun remains at the same location through different eccentricities.[2] To determine the orbits, we need to know the semi-major and semi-minor axis distances. We can assume that the semi-major axis of Earth's orbit remains the same.[3] We know that currently, with $e = .0167$,

$$r_p = 9.1402640 \times 10^7 \quad , \quad r_a = 9.4509460 \times 10^7.$$

The relation between the semi-major axis and the perihelion or aphelion is $r_p = (1 - e)a$ and $r_a = (1 + e)a$, respectively, where a is the length of the semi-major axis. Thus, $a = r_p/(1 - e)$ or $a = r_a/(1 + e)$. We get that the Earth's semi-major axis is

$$a = 9.2954988 \times 10^7 .$$

2. The sun actually exhibits slight orbital movement due to the Earth's gravitational force, about 1% on average. However, for simplicity, we assume this amount to be neglible.

3. See article on "Milankovitch cycles" in Wikipedia for reference.

Combined with the relation between the semi-minor axis, b, and eccentricity, i.e. $b/a = \sqrt{1-e^2}$, we get that

$$b \;=\; 9.2942025 \times 10^7.$$

To plot the current orbit versus a purely circular orbit, for reference we choose the Sun as the origin in our 2D graph. The Sun is in fact one of the foci of the elliptical orbit. For a purely circular orbit the foci and the center are the same. In this case, the radius is equal to the semi-major axis, a, with the Sun at the origin. In the case with eccentricity .0167, we need to determine where the center of the ellipse is, relative to the Sun placed at the origin. The distance from the center of the ellipse to either foci is given by $a \times e$. Thus, the current orbit of the Earth is an ellipse with major and minor axis given by a and b, respecitively, centered at $-ae$. We illustrate this in the following graph.

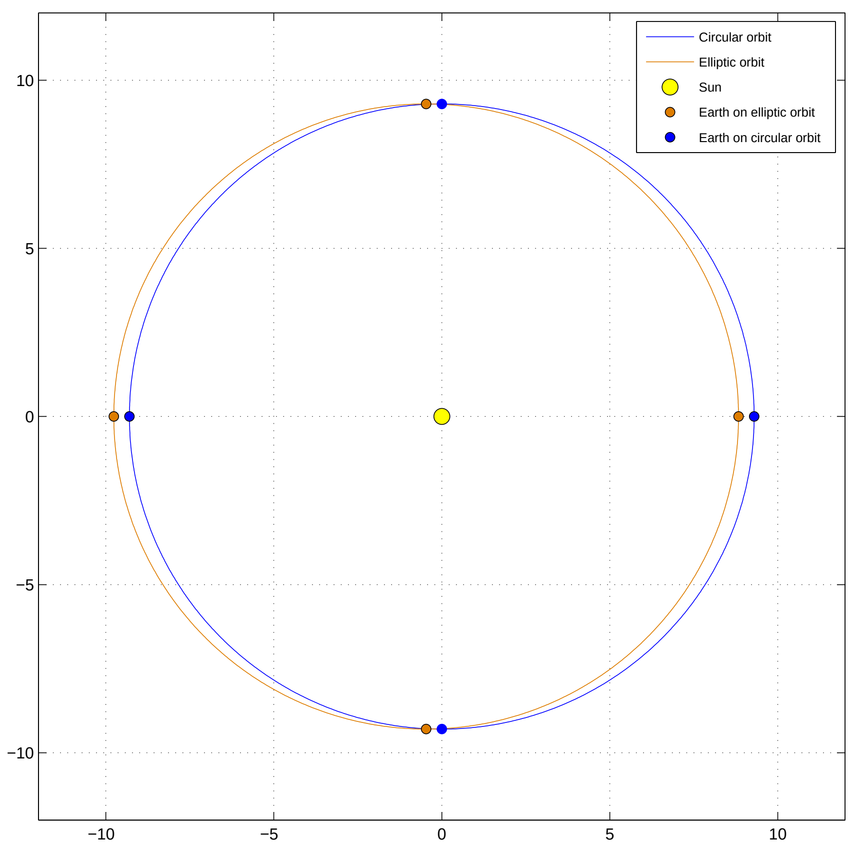

Figure 3. Note that the Earth on the elliptic orbit (orange trajectory above) at the aphelion (left most orange circle) is outside of the circular orbit, while the Earth on the elliptic orbit at the perihelion (rightmost orange circle) is inside the circular orbit. Also, note that the orbit trajectories are drawn to scale and are not representative of the true magnitudes.

One final note, under the assumptions of our model (in particular, the Sun's movement is negligible) and Kepler's second law ("A line joining a planet and the Sun sweeps out equal intervals of time"), the total radiation levels per annum by the Sun on the Earth is negligible for eccentricity.[4]

4. Again, see Wiki article on "Milankovitch cycles".

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