Appendix 1 Theoretic Formulation

Music is the Finite Union of Notes and Intervals as a Composite Function of the Fundamental Pitch-Position Algorithm so that the Binary Path of Tonal Movement is 3-fold not 2-fold as Commonly –Thought

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Music is a re-iterative composition by the finite union of notes and open intervals added to the fundamental tone, which is created through a pitch-position intonation algorithm in which every possible tonal value function, relation, and element in the system is determined by the affine projection of the R:Z vector connecting the monotonic function of pitch to the barycenter of the tone values vector stack.

To make this concept more rigorous let us define the fundamental x and function sets {f(x)} as open intervals and {g(x)} as notes which are secondary centers called keys, that act as additional centers upon which new tonal constructs can be built in a binary walk of notes and intervals.

The musical topos M =(x, {f(x)}, {g(x)}) a graph of functions f(x) which is a tone value triangle (line) in the form TV(1) + TV(2) + C = 0 which is called a cipher because the value of the linear function is 0, while g(x) > 0 is a greater than function that makes the scale sequence a partially ordered set in which each note is greater than the secondary center, just as each secondary center note is itself formed by adding to the fundamental.

Since F(x) and g(x) are van den Dries semi-algebraic functions, the topology of musical topos M is completely determined.

Perhaps the most characteristic aesthetic in music is the orthonormality of the Hilbert space created by the tone values vector stack resulting from the metric of independent scale sequences, which is extra-ordinary because any 2 tonal values vectors (not the same) are always equal and independent so that every angle is perpendicular and every distance is 1.

Since every tone value element satisfies $a = a^2$, it follows that the Pythagorean distance formula becomes is $a + b = c$ and that $1 + 1 = 1$ is the equation for both a pitch-position unit triangle, equilateral triangle with sides of 1, and a unit sphere with radius 1 where 1 is defined by 1 log cycle.

The tone values vector space is highly ordered but still undefined without a connection between the barycenter of the tone values space and the monotonic function of pitch by an R:Z cipher vector whose properties are extended to the entire vector stack through the barycenter. This principle is well-known in music: a note is undefined without a key.

Therefore a musical key is undefined without a fundamental. Trying to define a key in the absence of a fundamental is analogous to trying to read time by the minute hand of the clock, with the hour hand or even the same center as the hour hand. It cannot be however that there are two sequences, chromatic and harmonic, in the same key but with

different centers (that is without the 2 paths sharing the same first notes in the sequence).

In the Hilbert Space tonality is expressed by tonal movements that are not perceived directly on the sound horizon where we hear the harmonic image as formed by addition to the secondary tonal center of the musical key when the tonality of the music is actually a function of the fundamental.

A musical system can have only 1 fundamental tone, which is not defined by a pitch value as commonly thought, nor by a position value (string 1, fret 0) but instead by the union of a pitch value to a position.

The union of pitch and position is a marriage by an intonation algorithm in where the mass, tension, and length of a vibrating string (or other oscillator parameters) are adjusted to connect pitch and position by an R:Z cipher function according the tuning rules. In this way the pitch-position line is precisely the tone point. (Line-point duality principle in geometry.)

This model for music topos M then describes a curious duality illusion in which the tonality of music is the composite function f x g but the harmonic message is embedded as an image in tone values space by the line of tonal projection which is precisely the vector between the logic and algebraic sub

lattices of the music topos M. Therefore tonality is the algebraic closure operator of the pitch-position algorithm and harmony is the image if the harmonic position encode by the pitch-position algorithm as an independent Abelian pair (tonality, harmony).

The first-order illusion is then created where by the listener cannot understand the path of tone movement in higher dimensions, but the guitarist can see and understand an exquisite geometry in higher dimension than are commonly recognized.

This is a versatile model for music since the tonal vector pair ({f(x)}, {g(x)}) can be (pitch, position), (string, fret), (string, fundamental), (algebra, logic), (harmony, melody), (syntax, semantics), (continuous, discrete), (originality, expressivity), and a number of other independent pairs. The sum total of all these orthogonal vector pairs determines the cultural significance of music.

Since the binary path of tonal movement in music is by function f, by function g, and by f x g in composition, I have now proven that the symmetry of tonality is SO3 and not planar as commonly thought.

Furthermore pitch value space without an R:Z cipher is not defined and cannot possibly be a toroidal manifold (which implies 2 fundamentals when there can be at most 1).

The real problem with the musical torus is that it does not allow additional tonal centers to be added to the system to form a multi-centric tuning.

I therefore assert here that to be defined a musical system must have at least 2 centers, but at most 1 fundamental.