# Earth-Mars Interplanetary Transport System

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The issue for this Zomb is to investigate various options for a transfer station which would allow a comfortable situation for rapid Earth-Mars and Mars-Earth transport.

The general approach to this problem will be to have a waystation near Earth, a waystation near Mars, and a ferry travelling on an orbit connecting the two waystations. In an ideal case, all three of these would be built by moving small asteroids into suitable orbits. Such a configuration alleviates the problems of cosmic and solar radiation during flight (build underground) and the problem of long periods in zero-g (build inside large rotating cylinders). There would also be room and kinetic energy to travel in (relative) comfort while carrying along enormous quantities of cargo.

## Approximations

We will make many approximations in this project to simplify the calculations. The following will always be made:

- Earth's orbit is circular.

- Earth and Mars have coplanar orbits.
- Transfer orbits are solutions to the restricted two-body problem.
- Orbital perturbations are unimportant.

Earth's orbit is only about 3% out of round, which does not affect anything in the analysis significantly. Earth's and Mars' orbits only miss being coplanar by 1.85 degrees, so assuming coplanarity simplifies the orbit calculations without qualitatively changing the results. In the restricted two-body problem, the primary gravitating body (the Sun, the Earth, or Mars) is taken to have far more mass than does the ferry or waystations. The result is that the primary gravitating body can be considered to be fixed in space. Finally, orbital perturbations will eventually have to be taken into account, but in a permanent system will probably be compensated by a low-thrust engine.

## The Hohmann Transfer Orbit

The traditional low-energy transfer orbit is the Hohmann orbit. For the Earth-Mars case, the Hohmann orbit is that which starts tangent to Earth's orbit at the initial position of Earth, and ends tangent to Mars' orbit at Mars. The Sun is always at one of the foci of the Hohmann ellipse (Kepler's First Law of Orbital Mechanics).

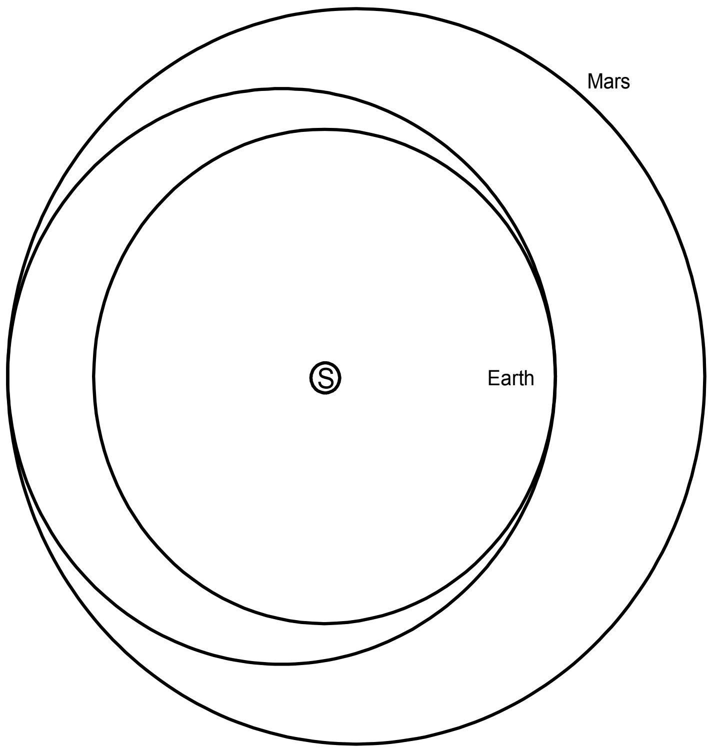


Figure 1. The Hohmann transfer orbit between Earth and Mars perihelion.

Let's analyze the Hohmann orbit that connects the Earth with Mars at Perihelion. The semi-major axis a of the Hohmann ellipse is one-half the sum of the Sun-Earth distance and the Sun Mars perihelion distance, or $1.785 \times 10^8$ km.

The orbital period τ is given by Kepler's Third Law of Orbital Mechanics as equal to:

$$\tau = 2\pi \left(\frac{a^3}{\mu}\right)^{1/2}, \qquad (1)$$

where μ is the Sun's gravitational constant $\mu = 1.372\times10^{11}\ km^3s^{-2}$. The orbital period is $\tau = 4.045\times10^{7}$ seconds, or about 1.28 years. {When the term "years" is used alone, Earth years is meant. Their length in seconds is $3.156\times10^{7}$ sec.} For completeness, the eccentricity of the Hohmann orbit is e = 0.152, and the foci are located $2.7\times10^{7}$ km from the center of the ellipse. From Figure 1 you can see that the transfer time is half the orbital period, or 0.64 years.

To determine how much energy is required to transfer from Earth's orbit the Hohmann transfer orbit, we calculate the difference between the planetary orbital velocity and the Hohmann orbital velocities where the two meet. Earth's orbital speed is 29.78 km/sec, and Mars' orbital speed at perihelion is 26.50 km/sec.

The speed in an elliptical orbit is

$$\tau = 2\pi \left(\frac{a^3}{\mu}\right)^{1/2},$$

(2)

where r is the distance between a point on the orbit and the Sun. The semi-major axis a of the transfer orbit at Mars perihelion is $1.785 \times 10^8$ km. The Earth-Sun distance is $1.50 \times 10^8$ km, and the Mars-Sun distance is $2.067 \times 10^8$ km at perihelion. Substituting these numbers into Eq. 2 gives the Hohmann orbital velocity at the Earth as 32.57 km/sec, and that at Mars as 23.64 km/sec. These are compared in the following table:

| | | |
|---|---|---|
| Earth Orbital Velocity 29.78 km/sec | Hohmann Velocity at Earth – 32.57 km/sec | Delta-V Earth to Hohmann - +2.79 km/sec |
| Mars Orbital Velocity 26.50 km/sec | Hohmann Velocity at Mars – 23.64 km/sec | Delta-V Hohmann to Mars - +2.86 |

These results ignore the gravitational effects of leaving and approaching massive planets. However, if one is only interested in the Hohmann transfer orbit, the total $\Delta V$ is about +5.65 km/sec, about half of Earth's escape velocity, and hence requiring about $\frac{1}{4}$ of the kinetic energy required to escape from Earth.

The problem with putting a ferry into a permanent Hohmann transfer orbit between Earth and Mars is that the ferry's orbital period is not equal to that of either Earth or Mars. The ferry would shuttle between the two orbits, but the planets would generally not be at the points where the Hohmann orbit is tangent to the orbits of the planets. The amount of energy needed to meet up with a Hohmann ferry is generally quite large. Accordingly, this is not a reasonable option.

Another possibility is to set the ferry in a 'resonant' orbit, so that the orbital period of the ferry is a rational fraction of Mars' orbital period. Remember, though, that the orbital period is a function of the semi-major axis. Accordingly, if the fraction is less than 1, the orbit will not be large enough to reach between Mars and Earth.

If the fraction is larger than 1, the orbit can be sufficiently large to intersect both Earth's and Mars' orbits. For example, a ferry orbit with a period of 3/2 Mars' orbital period would match up with Mars in its orbit every 3 Mars orbits. Having a large enough number of ferries would allow transport quite often. However, when the fraction is larger than 1, the velocity required to match both Earth's and Mars' orbital velocity is quite large, and becomes larger with the eccentricity of the ferry's orbit. In the end, this option is not a practical option.

Lagrange Points and the Interplanetary Transport Network

We want to set up our Earth-Mars transit system using semi-permanent waystations and ferries. One particularly effective approach is to place the waystation for Earth at the Sun-Earth Lagrange point 1 or 2 (SEL 1 or SEL2), and the Mars waystation at the Sun-Mars Lagrange point 1 or 2 (AML1 or SML2). The Lagrange points in the restricted three-body problem (2 massive bodies, one weightless body) are critical points of the total gravitational potentials of the Sun and a planet. The L1 and L2 points have an orbital period equal to that of the associated planet.

SEL1 and 2 are about $\pm 1.5 \times 10^6$ km from Earth along the Earth-Sun line, while SML1 and 2 are about $\pm 10^6$ km from Mars along the Mars-Sun Line. Such points can be stable, semi-stable, or unstable. The L1 and L2 points are semistable saddle-shaped critical points, so that the gravitational force relative to the Lagrange points radial to the Sun act to repel from the point, while the gravitational force perpendicular to the Sun attracts toward the point.

$$\tau = 2\pi \left( \frac{a^3}{\mu} \right)^{1/2}$$

Figure 2. A contour plot of the effective potential due to gravity and centrifugal force showing the Lagrange points and their potential gradients.

We can use the anomalous orbital velocities of the Lagrange points to help match velocities at each end of the Earth-Mars journey. Let's recalculate the Hohmann transfer orbit with the initial point being Earth's L2 point and the final point being Mars' L1 point.

The semi-major axis of a Hohmann transfer orbit between SEL2 to SML1 is $1.7775 \times 10^8$ km, and the radius of SEL2 from the Sun is $1.515 \times 10^8$ km. The transfer orbital velocity is thus 31.71 km/sec at SEL2. The orbital velocity of SEL2 is larger than that of the Earth by 1%, or 30.08 km/sec. The velocity needed to transfer from SEL2 to the Hohmann transfer orbit is thus +1.63 km/sec, compared to 2.28 km/sec when the Hohmann transfer orbit is plotted between planetary orbits.

The radius of SML1 from the Sun is $2.057 \times 10^8$ km. The corresponding Hohmann orbital velocity is 23.32 km/sec. The orbital velocity of SML1 is about 0.5% smaller than that of Mars itself, or 26.37 km/sec. The delta-V for the transfer between orbits at Mars is now +3.05 km/sec, compared to 3.22 km/sec.

The net saving of energy from plotting the Hohmann orbit between SEL2 and SML1 is thus about 0.82 km/sec.

The other reason for having waystations at SEL2 and SML1 is that these points are gateways to the Interplanetary Transport Network (ITN). The ITN is a network of low-energy pathways connecting planets and moons in the Solar System. These pathways allow one to transfer between most bodies in the Solar System using very small expenditures of fuel, but involve rather long transit times. Still, they are useful for launch of deep-space probes and for moving large amounts of stable cargo.

The ITN trajectories also allow transfer between orbits with far less $\Delta V$ than required by a Hohmann orbit, while requiring similar, and in some cases, shorter periods of time. As an example of what can be done by launching from SEL2, a ferry given a ΔV of 2.103 km/sec from SEL2 will arrive at Mars roughly 8 months later with the right velocity for injection into a low Mars orbit. This is compared with a Hohmann orbit (ignoring escaping from Earth's gravitational field) requiring ΔV = 5.65 km/sec. This is just a terribly non-optimized case – my simulation program is not well designed for tackling qualitative dynamics problems. However, adjusting the velocity a small amount yields launch points suitable for a wide range of initial Earth-Mars configurations.

In summary, the idea of an Earth-Mars ferry is difficult to accomplish owing to the requirement for rather large matching velocities. A more practical approach may lie in transferring large ships between Solar System Lagrange points. The ships would not be in a perpetual orbit, but would rest between launches at a Lagrange point of the appropriate planet. Their mass never requires large application of delta-V, with the result that movement of cargo and people within the inner Solar System can be treated as a routine and relatively inexpensive process. This approach offers many of the benefits of a permanently orbiting ferry, while avoiding the excessive delta-V requirements of that scheme.